METHOD FOR ON-DEMAND GENERATION OF INDIVIDUAL RANDOM NUMBERS

OF A SEQUENCE OF RANDOM NUMBERS OF A 1/F NOISE

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Cross-Reference to Related Application:

This application is a continuation of copending International

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designated the United States and was not published in English.

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Background of the Invention:

Field of the Invention:

The invention relates to a method for generating sequences of

random numbers of a 1/f noise.

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Random numbers of a 1/f noise can be used, for example, in a

transient circuit simulation that takes account of noise

influences. The 1/f noise is understood as a stochastic

process with a specific frequency spectrum that can be

20   described by the equation

$$S(f) \propto \frac{1}{f^{\beta}}, \beta \in \left]0,1\right[.$$

1/f noise sources are suitable for modeling noise influences

25   in a multiplicity of technical and physical systems, and for

systems for estimating and forecasting events on the financial

markets. In particular, many electronic components such as,

for example, pn diodes and MOS field-effect transistors

exhibit 1/f noise sources.

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It is possible to approximate 1/f noise sources by carrying

out a summation of the effects of many noise sources which

each exhibit a Lorentz-spectrum as a frequency spectrum. Such

noise sources can be modeled, for example, by the system

10   response of a linear time-invariant system, which can also be

denoted as an LTI-system, at whose system input white noise is

applied. It is disadvantageous in this mode of procedure for

the dimension of the system of differential equations that are

solved numerically to be excessively inflated. The result of

15   this is long computing times and a high storage requirement of

a computer system that is used to simulate a system that is

subject to the influence of 1/f noise.

Summary of the Invention:

20   It is accordingly an object of the invention to provide a

method for on-demand generation of individual random numbers

of a sequence of random numbers of 1/f noise that overcomes

the above-mentioned disadvantages of the prior art methods of

this general type, which can be carried out quickly and with a

25   low computational outlay. It is also an object of the

invention to specify an improved method for simulating a

technical system that is subject to 1/f noise. Finally, the aim is also to specify a computer system having a computer program for determining sequences of random numbers of 1/f noise that can be executed quickly and that lays claim to only

5   a few resources of a computer system.

In accordance with the invention, the problem of noise simulation in the modeling of the system to be simulated is transformed into the problem of generating a random number

10 sequence. In accordance with the invention, the correlations of the random numbers are determined, and this is used for a simple and accurate generation of the corresponding random number sequences.

15 The method according to the invention for generating at least one sequence of random numbers of 1/f noise first provides the following steps in this case:

a) determining a desired spectral value β; and

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b) determining an intensity constant const.

Characteristics of the 1/f noise to be simulated are thereby fixed.

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As a result, the number of random numbers of the 1/f noise that are to be generated, and a starting value for a running variable n used for the simulation are fixed.

5   Until the desired number of elements y(n) of one or more vectors $\underline{y}$ of length n are calculated from 1/f-distributed random numbers, the invention provides the loop-type repetition of the following steps:

10   a) increasing a current value of a running variable n by 1;

b) fixing a simulation time step [t_{n-1}; t_n];

15   c) determining an element $\underline{C}_{ij}$ of a covariance matrix $\underline{\underline{C}}$ of dimension (n × n) according to:

$$\underline{\underline{C}}_{ij} := const \cdot \left(-\left|t_j - t_i\right|^{\beta+1} + \left|t_{j-1} - t_i\right|^{\beta+1} + \left|t_j - t_{i-1}\right|^{\beta+1} - \left|t_{j-1} - t_{i-1}\right|^{\beta+1}\right),$$
$$i, j = 1, ..., n$$

20   d) determining a matrix $\underline{\underline{C}}^{-1}$ by inverting the covariance matrix $\underline{\underline{C}}$;

e) determining a variable σ in accordance with the rule

$$\sigma = sqrt(1 / e(n,n)),$$

where sqrt denotes the "square root" function, and $e(n,n)$ denoting the element of the inverted covariance matrix $\underline{\underline{C}}^{-1}$ indexed by (n,n);

5   f) determining a $(0,1)$-normally distributed random number which forms the nth component of a vector $\underline{x}$ of length n;

g) forming a variable μ from the first $(n-1)$ components of the nth row of the inverted covariance matrix $\underline{\underline{C}}^{-1}$ and the $(n-1)$

10   elements of the vector $\underline{y}$ which were calculated for a preceding $(n-1)$ simulation time step, specifically in accordance with the following rule:

$$\mu := -\frac{y_{(n-1)}^{T} \cdot \underline{\underline{C}}^{-1}_{\bullet,n}}{\underline{\underline{C}}^{-1}_{n,n}}$$

$y_{(n-1)}$ denoting the first $(n-1)$ components of the vector $\underline{y}$,

15      $\underline{\underline{C}}^{-1}_{\bullet,n}$ denoting the first $(n-1)$ components of the nth row of the inverted covariance matrix $\underline{\underline{C}}^{-1}$, and $\underline{\underline{C}}^{-1}_{n,n}$ denoting the element of the inverted covariance matrix $\underline{\underline{C}}^{-1}$ indexed by (n,n); and

20 h) calculating an element $y(n)$ of the vector $\underline{y}$ of length n from 1/f-distributed random numbers according to the following rule:

$$Y(n) = x(n) * \sigma + \mu.$$

Simulations of technical systems can be arbitrarily lengthened with the aid of the method according to the invention. It is possible for this purpose to generate additional 1/f-distributed random numbers in a simple way when already

5   generated 1/f-distributed random numbers are to hand. Moreover, a simulation can be based on results of previously simulated time intervals. This so-called restart capability constitutes a very important property for the practice of simulation. It is precisely for 1/f noise sources that this

10   can be achieved only with difficulty, because random numbers that simulate a 1/f noise source for a specific time interval depend on random numbers for earlier time intervals that have already been determined numerically. The present invention also permits the use of an adaptive step size control without

15   thereby significantly increasing the computation times for the simulation of the technical system. Such an adaptive step size control considerably increases the precision and the computation time efficiency in the numerical determination of the dynamic performance of a simulated technical system.

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It is no longer necessary in the case of the method according to the invention to prescribe the time interval to be simulated. Precisely by the provision of variable step sizes, it is also possible to perform an adaptation to current system

25   dynamics, and this increases the accuracy of the simulations.

The present invention specifies a method for generating

sequences of 1/f-distributed random numbers successively, that

is to say element by element. In this case, the method

ensures that each newly generated random number depends in a

5   correct way in the stochastic sense on the previously

generated 1/f-distributed random numbers. It is thereby

possible to generate the respectively required random numbers

in the course of the numerical simulation of a circuit.

10   The invention uses the theory of conditional probability

density in order to generate a 1/f-distributed random number

that correctly ensures the stochastic relationship of the

random number with the already generated random numbers

required for preceding simulation steps.

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In a particularly advantageous refinement of the method

according to the invention, q sequences of random numbers of a

1/f noise are calculated simultaneously, instead of the steps

to be repeated in the loop-type fashion:

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a) determining a (0,1)-normally distributed random number

which forms the nth component of a vector $\underline{x}$ of length n;

b) forming a variable μ from the first (n-1) components of the

25   nth row of the inverted covariance matrix $\underline{\underline{C}}^{-1}$ and the (n-1)

elements of the vector $\underline{y}$ which were calculated for a preceding

(n-1) simulation time step, specifically in accordance with the following rule:

$$\mu := -\frac{y_{(n-1)}{}^{T} \cdot \underline{\underline{C}}^{-1}_{\bullet,n}}{\underline{\underline{C}}^{-1}_{n,n}}$$

where $y_{(n-1)}$ denotes the first (n-1) components of the vector $\underline{y}$,

5   $\underline{\underline{C}}^{-1}_{\bullet,n}$ denotes the first (n-1) components of the nth row of the

inverted covariance matrix $\underline{\underline{C}}^{-1}$, and $\underline{\underline{C}}^{-1}_{n,n}$ denotes the element of

the inverted covariance matrix $\underline{\underline{C}}^{-1}$ indexed by (n,n); and

c) calculating an element y(n) of a vector $\underline{y}$ of length n from

10   1/f-distributed random numbers according to the following

rule:

$$Y(n) = x(n) * \sigma + \mu,$$

15   the following steps are provided:

d) determining a quantity q of (0,1)-normally distributed

random numbers $x_{k,n}$ which form the respective last component of

the vectors $\underline{x}_k$ of length n, k = 1, ..., q. It is to be borne

20   in mind in this case that the respective first (n-1)

components of the vectors $\underline{x}_k$ have already been calculated

previously in the step.

e) forming q variables μ_k in accordance with the following rule:

$$\mu_k := -\frac{y_{(n-1),k}^{T} \cdot \underline{\underline{C}}_{\bullet,n}^{-1}}{\underline{\underline{C}}_{n,n}^{-1}}$$

$y_{(n-1),k}$ denotes the first (n-1) components of the vector $\underline{y}_k$ that

5   were calculated for a preceding simulation time step. $\underline{\underline{C}}_{\bullet,n}^{-1}$

denotes the first (n-1) components of the nth row of the

inverted covariance matrix $\underline{\underline{C}}^{-1}$ and $\underline{\underline{C}}_{n,n}^{-1}$ denotes the element of

the inverted covariance matrix $\underline{\underline{C}}^{-1}$ indexed by (n,n). This is

carried out for k = 1, ..., q.

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f) calculating q elements $y_{k,n}$ which form the respective nth

component of the vector $\underline{y}_k$ of length n from 1/f-distributed

random numbers, specifically according to the following rule:

15                         $y_{k,n} = x_{k,n} * \sigma + \mu_k,$

    k = 1, ..., q.

The q vectors $\underline{y}_k$ (k = 1, ..., q) of length n from 1/f-

20   distributed random numbers are arranged with particular

advantage in a NOISE matrix and specify in a simulation the

1/f noise influences of a system to be simulated.

In accordance with the invention, the concept for simulating
1/f noise is based according to the invention on the following
train of thought. The dynamic performance of a system that is
exposed to stochastic influences is modeled adequately by a

5   stochastic process. In order to simulate such a system
dynamics, the general approach is to calculate numerically
individual random realizations (so-called paths) of the
fundamental stochastic processes. Simulating systems with 1/f
noise sources requires paths of stochastic integrals of the

10   form $\int_0^t Y(s)\eta_{\frac{1}{f}}(s)ds$ to be calculated numerically. Here, s is an

integration variable and t (upper integration limit) denotes
the time, $\eta_{\frac{1}{f}}(s)ds$ denotes a 1/f noise source, and Y(s) a

stochastic process that describes the temporal dynamics of a
variable, for example the electric voltage in the circuit

15   simulation.

Using $B_{FBM}(s)$ to denote that stochastic process whose
derivative (mathematically: derivative in the distribution
sense) yields the 1/f noise process $\eta_{\frac{1}{f}}(s)$, it is possible to

20   write the stochastic integral to be calculated as

$$\int_0^t Y(s)\eta_{\frac{1}{f}}(s)ds = \int_0^t Y(s)dB_{FBM}(s) \quad (1.1)$$

The integral on the right-hand side is to be interpreted as a Riemann-Stieltjes integral of the stochastic process Y(s) with the process $B_{FBM}(s)$ as an integrator. This integral can be approximated by a sum by decomposing the integration interval

5   [0,t] in accordance with $0 \equiv t_0 < t_1 < \ldots < t \equiv t$ into n disjoint subintervals $[t_i, t_{i-j}]$, i=1, ..., n:

$$\int_0^t Y(s)dB_{FBM}(s) \approx \sum_{[illegible]}^{n} Y(t_{i-1})[B_{FBM}(t_i) - B_{FBM}(t_{i-1})] \quad (1.2)$$

10 This sum is a random variable. The dependence on the result ω of the random experiment has been consistently omitted.

A process $B_{FBM}(s)$ whose generalized derivative has a 1/f spectrum is known in the literature under the name of

15 'Fractional Brownian Motion'. $B_{FBM}(s)$ is a Gaussian stochastic process and is completely characterized as such by its expectation

$$E(B_{FBM}(s)) = 0 \; \forall \, s \in R \quad (1.3)$$

20

and by its covariance function

$$Cov(B_{FBM}(s), B_{FBM}(t)) = const \cdot \left(|s|^{\beta+1} + |t|^{\beta+1} - |t-s|^{\beta+1} \right) \quad (1.4)$$

The method according to the invention for on-demand generation
of suitable random numbers essentially reduces the simulation
of 1/f noise influences to the generation of realizations of
the random variables $[B_{FBM}(t_i) - B_{FBM}(t_{i-1})]$, that is to say to

5 the generation of increments of Fractional Brownian Motion.

The present invention permits the online generation of the
required realizations of the random variables: $\Delta B_{FBM}(i)$, that
is to say in the course of the successive integration of the

10 system equations. This results in two requirements placed on
the method:

(a)   The length n of the sequence of random numbers
$\left\{\Delta B_{FBM}(1),...,\Delta B_{FBM}(n)\right\}$ must remain variable during a simulation run.

15 In particular, it must be possible at any time to lengthen the
simulation (restart capability). This implies the capability
of the method to generate the additional random numbers
required for this purpose such that they are correlated in a
correct way with the subsequence already generated.

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(b)   Let t_i be the currently attained time in the course of a
simulation. It must then be possible to determine the time
interval $[t_i, t_{i+1}]$, that is to say the step size of the next
integration step, from the instantaneous system dynamics -

25 that is to say adaptively.

The invention meets both requirements by specifying a rule as to how a realization of $\{\Delta B_{FBM}(1),...,\Delta B_{FBM}(n)\}$, that is to say a sequence of random numbers, can be generated successively,

5   that is to say element by element. The step size $\Delta t_i := t_i - t_{i-1}$ can be freely selected in this case for each new random number.

The first step is to investigate the approach for so-called
10   "conditional densities".

The first step is to consider the distribution of the random variable vectors $(\Delta B_{FBM}(1),...,\Delta B_{FBM}(n))$.

15   Since the individual random variables $\Delta B_{FBM}(i)$ represent increments of a Gaussian stochastic process, the random variable vector $(\Delta B_{FBM}(1),...,\Delta B_{FBM}(n))$ determines an n-dimensional Gaussian-distributed random variable, and thus completely defined by its (n-dimensional) expectation $\mathbf{E}$ and its
20   covariance matrix $\underset{=}{C}$. The two variables can be calculated from the formulas (1.3) and (1.4) as

$$E\big(\Delta B_{FBM}(i)\big) = 0, i = 1,...,n \qquad (3.5)$$

$$\underline{\underline{C}}_{i,j} := Cov\big(\Delta B_{FBM}(i), \Delta B_{FBM}(j)\big) = const \cdot \Big(-\big|t_j - t_i\big|^{\beta+1} + \big|t_{j-1} - t_i\big|^{\beta+1}$$
$$+ \big|t_j - t_{i-1}\big|^{\beta+1} - \big|t_{j-1} - t_{i-1}\big|^{\beta+1}\Big) \qquad (3.6)$$
$$i,j = 1,...,n$$

The online method according to the invention is now to be

specified in the form of a complete induction.

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The basis of the induction - and thus the starting point of

the method - is the realization of a real Gaussian

distribution with expectation 0 and variance $\sum_{11} = 2 \cdot const \cdot \big|\Delta t_1\big|^{\beta+1}$.

10 It must be specified for the purpose of inductive influence

how to expand a realization of $\big(\Delta B_{FBM}(1),...,\Delta B_{FBM}(n-1)\big)$ by a

realization of $\Delta B_{FBM}(n)$ so as to produce overall a realization

of $\big(\Delta B_{FBM}(1),...,\Delta B_{FBM}(n)\big)$. In order to simplify the notation, the

already "cubed" subsequence of random numbers may be denoted

15 by $(y_1, \ldots, y_{n-1}) =: y_{(n-1)}^T$ and the realization, still to be cubed,

of $\Delta B_{FBM}(n)$ may be denoted by y_n.

The problem can now be formulated as now described.

20 Let there be given an n-dimensional zero mean Gaussian random

variable Z with the covariance matrix $\underline{\underline{C}}$. Let the first $n-1$

elements of a realization of Z be already cubed and known in

the form of a random number vector $\underline{y}_{(n-1)}$.

The aim is now to find the distribution from which the nth element y_n must be taken in order to complete $\underline{y}_{(n-1)}$ to form a realization $\underline{y} = (\underline{y}_{(n-1)}, \underline{y}_n)$ of **Z**.

5

A solution to this task can be found when the conditional probability density $f\left(y_n \middle| y_{(n-1)}\right)$ are considered for y_n - with the condition that $y_{(n-1)}$ already holds. This variable can be calculated in the present case of a Gaussian normal

10   distribution as

$$f\left(y_n \middle| y_{(n-1)}\right) = \frac{1}{\sqrt{2\pi \dfrac{1}{\underline{\underline{C}}_{n,n}^{-1}}}} \exp\left\{ -\frac{1}{2\dfrac{1}{\underline{\underline{C}}_{n,n}^{-1}}} (y_n - \mu)^2 \right\}. \quad (3.7)$$

The variable $\underline{\underline{C}}_{n,n}^{-1}$ is yielded in this case from the following

15   notation of the inverted covariance matrix $\underline{\underline{C}}^{-1}$ as:

$$\underline{\underline{C}}^{-1} = \left(\begin{array}{c|c} \underline{\underline{C}}_{(n-1)}^{-1} & \underline{\underline{C}}_{\bullet,n}^{-1} \\ \hline \left(\underline{\underline{C}}_{\bullet,n}^{-1} \right) & \underline{\underline{C}}_{n,n}^{-1} \end{array} \right), \quad (3.8)$$

where $\underline{\underline{C}}_{(n-1)}^{-1} \in R^{(n-1)X(n-1)}$, where $\underline{\underline{C}}_{\bullet,n}^{-1} \in R^{(n-1)}$ and where $\underline{\underline{C}}_{n,n}^{-1} \in R$.

20

The variable μ stands for

$$\mu := -\frac{\underline{y}_{(n-1)}{}^T \underline{\underline{C}}^{-1}_{\bullet,n}}{\underline{\underline{C}}^{-1}_{n,n}} , \qquad (3.9)$$

5   The conditional density $f\left(y_n \middle| y_{(n-1)}\right)$ is therefore the probability

density of a Gaussian normal distribution with a mean of μ and

variance of $\dfrac{1}{\underline{\underline{C}}^{-1}_{n,n}}$.

It must hold that $\underline{\underline{C}}^{-1}_{n,n} \neq 0$ in order for the above variance to

10   exist. This is ensured on the basis of the following

argumentation. Let $\underline{\underline{C}}$ and $\underline{\underline{C}}^{-1}$ have the same characteristic

directions and inverse eigenvalues. An eigenvalue 0 of the

matrix $\underline{\underline{C}}^{-1}$ would therefore have an infinite variance of the

random variable vector $\left(\Delta B_{FBM}(1),...,\Delta B_{FBM}(n)\right)$ as a consequence. It

15   may therefore be presupposed that all the eigenvalues of $\underline{\underline{C}}^{-1}$

are nonvanishing. Since the eigenvalues of $\underline{\underline{C}}^{-1}$ are non-

negative, in any case, it therefore holds that: The matrix

$\underline{\underline{C}}^{-1}$ is symmetrical and positively defined. It is possible by

renaming the coordinate axes to bring this matrix from the

20   form (3.8) to the following form:

$$\underline{\underline{\overline{C}}}^{-1} := \left(\begin{array}{c|c} \underline{\underline{\overline{C}}}^{-1}_{n,n} & \left(\underline{\underline{\overline{C}}}^{-1}_{\bullet,n}\right)^{T} \\ \hline \underline{\underline{\overline{C}}}^{-1}_{\bullet,n} & \underline{\underline{\overline{C}}}^{-1}_{(n-1)} \end{array} \right) \qquad (3.10)$$

This matrix is likewise symmetrical and positively defined per

constructionem. In accordance with the Sylvester criterion

5   for symmetrical and positively defined matrices, it follows

therefrom that $\left(\underline{\underline{\overline{C}}}^{-1}\right)_{11} = \underline{\underline{C}}^{-1}_{n,n} > 0$, and the assertion is demonstrated.

The method according to the invention reduces a simulation of

$\dfrac{1}{f}$ noise sources to the generation of Gaussian-distributed

10   random numbers.

The inverted covariance matrix $\underline{\underline{C}}^{-1}$ (an $n \times n$ matrix) is

required in order to generate a random number y_n that is

correlated in the way required with an already generated

15   sequence $y_{(n-1)}$. Strictly speaking, all that is required is

knowledge of the nth row of this matrix, that is to say

knowledge of $\left(\left(\underline{\underline{C}}^{-1}_{\bullet,n}\right)^{T}, \underline{\underline{C}}^{-1}_{n,n}\right)$. As is to be read from formula (3.6),

the covariance matrix $\underline{\underline{C}}$ depends on the decomposition of the

simulation interval $[0, t_n]$ into disjoint subintervals (step

20   sizes) $[t_{i-1}, t_i]$. In particular, the last column of $\underline{\underline{C}}$ depends

on t_n (doing so identically with the last row because of the

symmetry of $\underline{\underline{C}}$), and thus on the current step size $\Delta t_n = t_n - t_{n-1}$.

The left-hand upper $(n-1)\times(n-1)$ submatrix $\bar{\underline{C}}$ of the $n \times n$

covariance matrix $\underline{C}$ is precisely the covariance matrix for a

random number sequence of length $n-1$. This covariance matrix

5   had already to be determined and inverted for calculating $y_{(n-1)}$

or for calculating the last element (y_{n-1}). In order to

accelerate the method, it is therefore possible to have

recourse to an incremental method for matrix inversion, for

example by use of the Schur complement.

10

The invention is also implemented in a method for simulating a

technical system that is subject to 1/f noise. In this case,

random numbers that have been determined with the aid of a

method according to the invention are used in modeling and/or

15   fixing the variables present on the input channels of the

system.

Also provided is a computer system and/or a computer program

for determining sequences of random numbers of a 1/f noise, or

20   for executing the other methods according to the invention.

The invention is also implemented in a data medium having such

a computer program. Furthermore, the invention is implemented

in a method in which a computer program according to the

invention is downloaded from an electronic data network, such

as from the Internet, for example, on a computer connected to the data network.

Other features which are considered as characteristic for the
5   invention are set forth in the appended claims.

Although the invention is illustrated and described herein as embodied in a method for on-demand generation of individual random numbers of a sequence of random numbers of 1/f noise,
10   it is nevertheless not intended to be limited to the details shown, since various modifications and structural changes may be made therein without departing from the spirit of the invention and within the scope and range of equivalents of the claims.

15

The construction and method of operation of the invention, however, together with additional objects and advantages thereof will be best understood from the following description of specific embodiments when read in connection with the
20   accompanying drawings.

Brief Description of the Drawings:

Fig. 1 is a block diagram of a technical system to be simulated;

25

Fig. 2 is a structogram for determining sequences of random

numbers of 1/f noise;

Figs. 3A-3F show calculation examples for a first simulation

5   time step;

Figs. 4A-4F show calculation examples for a second simulation

time step; and

10   Figs. 5A-5F show calculation examples for a third simulation

time step.

Description of the Preferred Embodiments:

Referring now to the figures of the drawing in detail and

15   first, particularly, to Fig. 1 thereof, there is shown a

schematic diagram of a noisy system that is to be simulated.

The system is described by a system model 1 that is indexed as

a box and describes the system behavior. The system behavior

20   results from input channels 2, which are also designated as

vector INPUT and from output channels 3, which are also

designated as OUTPUT. Furthermore, a system-dictated noise is

provided, which is present on noise input channels 4 and which

is also designated as a vector or as a matrix NOISE. The

25   matrix NOISE is present when the noise is taken into account

with a plurality of channels, each column of the matrix NOISE

containing a vector of noise values which are present on a
noise input channel.

The noise on the noise input channels 4 is preferably

5   interpreted as a noise-dictated alteration of the system model

1.

The behavior of the input channels 2 and of the output
channels 3 can be described by a system of differential

10   equations or by a system of differential-algebraic equations,
so that reliable predictions of the system behavior are
possible.

For each time step of the simulation of the system shown in

15   Fig. 1, a vector OUTPUT of the output channels 3 is calculated
for a vector INPUT present on the input channels 2 and for a
vector NOISE present on the noise input channels 4.

For the simulation over a relatively long period of time, the

20   vectors INPUT, OUTPUT, NOISE are expediently specified as a
matrix, a respective column k of the relevant matrix
containing the values of the corresponding time series of the
relevant INPUT, OUTPUT, NOISE.

25   Fig. 2 illustrates how a respective vector y_k is attained which
forms a column k of the matrix NOISE for the noise input

channels 4 of the system model 1. Each vector $\underline{y}_k$ serves for simulating a noise source.

In a first step, a desired spectral value β, and an intensity

5   constant const are fixed. Furthermore, a counter n of the current simulation time interval is set to 0. The following sequence of computation steps is then carried out successively for each simulation time step.

10   First, the current simulation time step is fixed. Equivalently to this, it is also possible to fix the end of the current simulation time step, thereby producing the next point in time under consideration.

15   Afterward, the counter n of the current simulation time step is incremented by one.

The covariance matrix $\underline{\underline{C}}$ of dimension (n × n) is subsequently determined according to equation (3.6).

20

This is followed by the step of inverting the matrix $\underline{\underline{C}}$, for example by use of a Cholesky decomposition. In order to raise the efficiency, it is also possible in this case to appeal to the inverse matrix of the previous step, for example by using

25   Schur complement techniques.

Next, the variable σ is calculated from the formula

$$\sigma = \text{sqrt}(1\ /\ e(n,n)\ )$$

5

where sqrt denotes the "square root" function and where $e(n,n)$ denotes that element of the inverted covariance matrix $\underline{\underline{C}}^{-1}$ which is indexed by (n,n).

10 In addition, a value of a $(0,1)$-normally distributed random variable X_k is extracted and the vector $\underline{x}_k$ of the normally distributed random numbers is thereby supplemented. The extracted random number has the expectation of 0 and the variance of 1. This step is carried out for each noise source

15 to be simulated.

Furthermore, a variable μ_k is formed. It is formed from the first $(n-1)$ components of the nth row of the inverted covariance matrix $\underline{\underline{C}}^{-1}$ and from the sequence of $(n-1)$ 1/f-

20 distributed random numbers which were calculated for the preceding $(n-1)$ simulation time steps.

For this purpose, the procedure is in accordance with formula (3.9). This step is carried out for each noise source k to be

25 simulated.

Finally, that element of the matrix NOISE is calculated whose

column index k specifies the noise source to be simulated and

whose row index is equal to n. The current simulation time

5   step is denoted by this. The currently calculated element

$r(k,n)$ of the matrix NOISE represents a random number which,

together with the superjacent (n-1) elements of the same

column k of NOISE, forms a vector $\underline{y}_k$ of length n from 1/f-

distributed random numbers. The vector $\underline{y}_k$ serves for

10   simulating one of the noise sources for the first n simulation

time steps.

Each element y_k of the nth row of NOISE is then determined, on

the basis of equations (3.7)-(3.9), from the last random

15   number x_k of the vector x and the variables μ_x and σ, to be

precise according to the following rule:

$$y_k = x_k * \sigma + \mu_x.$$

20   Figs. 3A to 5F depict exemplary embodiments reproducing

concrete results of calculation.

The value of the spectral value β is always adopted in this

case as 0.5. The value of the intensity const is arbitrarily

25   adopted as 1.0. Three random numbers are simultaneously

processed in each case in accordance with the simulation of three noise sources simultaneously acting, on separate channels, on the system to be simulated and which are disposed in a vector $\underline{y}_k$, k being an integral value from 1 to 3.

5

Figs. 3A-3F show a calculation example for a first simulation time step $[t_0, t_1] = [0, 0.5]$.

Fig. 3A shows the covariance matrix $\underline{\underline{C}}$ of dimension 1 × 1 for

10 generating a 1/f-distributed random number in the case of the simulation step size. $\underline{\underline{C}}$ represents here only a scalar with the value 0.70, because $\underline{\underline{C}}(1,1)$ - that is to say with i=j=1 - results upon applying equation (3.6) as

15
$$1.0 \cdot \left(-\left| t_1 - t_1 \right|^{0.5+1} + \left| t_{1-1} - t_1 \right|^{0.5+1} + \left| t_1 - t_{1-1} \right|^{0.5+1} - \left| t_{1-1} - t_{1-1} \right|^{0.5+1} \right) =$$
$$0 \quad + \quad 0.5^{1.5} \quad + \quad 0.5^{1.5} \quad - \quad 0 \qquad = 0.707106\ldots;$$

Fig. 3B shows the inverse of the covariance matrix $\underline{\underline{C}}$ from Fig. 3A, something which was performed here by use of a Cholesky decomposition not illustrated in more detail here. A check of

20 $(\underline{\underline{C}} \, \underline{\underline{C}}^{-1}) = (0.707106 \ldots 0.707106 \ldots^{-1})$ yields the correct value 1, and this illustrates the correctness of the value for $\underline{\underline{C}}(1,1)$.

Fig. 3C shows a variable σ for the first simulation step n=1.
This is yielded from the equation

$$\sigma = \text{sqrt}(1\ /\ 0.707106...\ ),$$

5

sqrt denoting the square root and e(1,1) denoting the element
0.707106..., indexed by (1,1), of the inverted covariance
matrix $\underline{\underline{C}}^{-1}$.

10   Fig. 3D shows three values x_1, x_2, x_3 of a (0,1)-normally
distributed random variable X_k for one noise source to be
simulated in each case. These values form the first element
of one vector $\underline{x}_k$ each of the normally distributed random
numbers. The extracted random numbers have the expectation of

15   0 and the variance of 1.

Fig. 3E shows three variables μ_k for each of the three noise
sources to be simulated. The variable μ_k is yielded in
accordance with formula (3.9) from the first (n-1) components

20   of the nth row of the inverted covariance matrix $\underline{\underline{C}}^{-1}$ as well as
from the sequence of (n-1) 1/f-distributed random numbers that
were calculated for the preceding (n-1) simulation time steps.
In the first simulation step, these two vectors each have the
length 0. It therefore follows for all the variables μ_k in the

25   first simulation step that: $\mu_k = 0$.

Fig. 3F shows three vectors $\underline{y}_k$ of length 1 of 1/f-distributed random numbers that simulate the behavior of three 1/f-distributed noise sources of the first simulation time step

5   $[0, t_1] = [0, 0.5]$.   The matrix NOISE is yielded from the three vectors $\underline{y}_k$.   The value k is an integral value of from 1 to 3 in this case.   Each element y_k of the first row of NOISE is determined on the basis of equations (3.7)-(3.9) according to the following rule from the last random number x_k of the

10 associated vector $\underline{x}$ and the variables μ_κ and σ_1.   For example, the first element $\underline{y}_1(n=1)$ of the first vector $\underline{y}_1$ is calculated as follows:

$$\underline{y}_1(n=1) = \underline{x}_1(n=1) \ * \ \sigma + \mu_1 =$$

15 $$= -0.35... \ * \ 0.84... \ + \ 0.00... \ =$$

$$= -0.30... \ .$$

$\underline{y}_2(n=1)$ and $\underline{y}_3(n=1)$ are calculated by analogy herewith.

20 With the aid of Figs. 4A to 4F, there is shown a calculation example of a second simulation time step $[t_1, t_2] = [0.5, 0.75]$. The value n for the second simulation time step is always equal to 2.

Fig. 4A shows the covariance matrix $\underline{\underline{C}}$ of dimension (n × n) = 2 × 2, which is required for generating one further random number each per noise source. Together with the result in accordance with Fig. 3F, the random number thus newly

5   generated forms a vector $\underline{y}_k$ of length 2 from 1/f-distributed random numbers. A vector $\underline{y}_k$ is generated in this case for each noise source. The covariance matrix $\underline{\underline{C}}$ is determined in this case according to equation (3.6).

10   For example, this is carried out at the element $\underline{\underline{C}}(2,1)$ - that is to say with i=2 and j=1. From applying equation (3.6), $\underline{\underline{C}}(2,1)$ is yielded as

$$1.0 \cdot \left(-\left|t_1 - t_2\right|^{0.5+1} + \left|t_{1-1} - t_2\right|^{0.5+1} + \left|t_1 - t_{2-1}\right|^{0.5+1} - \left|t_{1-1} - t_{2-1}\right|^{0.5+1}\right) =$$
$$= \left(-\left|0.5 - 0.75\right|^{0.5+1} + \left|0 - 0.75\right|^{0.5+1} + \left|0.5 - 0.5\right|^{0.5+1} - \left|0 - 0.5\right|^{0.5+1}\right) =$$
$$-0.125 \quad + \quad 0.6495... \quad + \quad 0 \quad\quad - 0.3535... = 0.1709....$$

15

Fig. 4B shows the inverse of the covariance matrix $\underline{\underline{C}}$ from Fig. 4A. A check (not illustrated here) of the condition ($\underline{\underline{C}}\ \underline{\underline{C}}^{-1}$) yields a matrix of dimension 2 × 2, in the case of which the elements indexed by (1,1) and (2,2) are equal to 1. The other

20   elements have the value 0.

Fig. 4C shows a variable σ that is calculated from the inverted covariance matrix $\underline{\underline{C}}^{-1}$ of step 4B. The variable σ is yielded as

5
$$\sigma = sqrt(1 / e(n,n)) = sqrt(1 / e(2,2)) =$$
$$= sqrt(1 / 4.79...) = 0.45...;$$

sqrt denoting the square root, and $e(2,2)$ denoting the element, indexed by $(2,2)$ of the inverted covariance matrix $\underline{\underline{C}}^{-1}$

10   from Fig. 4B.

Fig. 4D shows three vectors $\underline{x}_k$ of independent $(0,1)$-normally distributed random numbers, the vectors $\underline{x}_k$ each having a length of 2. A $(0,1)$-normally distributed random variable x_k is

15   extracted per noise source to be simulated. The extracted random number in each case has the expectation of 0 and the variance of 1. The vectors $\underline{x}_k$ of the normally distributed random numbers from Fig. 3D are thereby supplemented such that the vectors $\underline{x}_k$ of the normally distributed random numbers from

20   Fig. 4D result.

Fig. 4E shows three variables μ_k that were calculated from the inverted covariance matrix $\underline{\underline{C}}^{-1}$ in accordance with step 4B and from the three random numbers in accordance with step 3F. For

25   each noise source to be simulated, the variable μ_k is

calculated from the (n-1) first components of the nth row of the inverted covariance matrix $\underline{\underline{C}}^{-1}$ and from the sequence of a quantity (n-1) of 1/f-distributed random numbers that were calculated in accordance with formula (3.9) for the preceding

5   (n-1) simulation time steps. In the second simulation step, the variable μ_k is thus calculated from the first component of the second row of $\underline{\underline{C}}^{-1}$ as well as from the first component of the vector $\underline{y}_k$. This is carried out, for example, with the aid of the value μ_1:

10

$$\mu_1 = \frac{\underline{y}^T_{(n-1),1} \cdot \underline{\underline{C}}^{-1}_{\bullet,n}}{\underline{\underline{C}}^{-1}_{n,n}} =$$

$$= \frac{-0.30... \cdot -1.15...}{4.79...} =$$

$$= -0.07....$$

Fig. 4F shows three vectors $\underline{y}_k$ of length 2 with 1/f-distributed random numbers that simulate the behavior of three 1/f-

15   distributed noise sources for the second simulation time step $[t_1, t_2] = [0.5, 0.75]$. The matrix NOISE is yielded from the three vectors $\underline{y}_k$. The value k is an integral value of from 1 to 3 in this case. Each element y_k of the second row of NOISE is determined on the basis of equations (3.7)-(3.9) according to

20   the following rule from the last random number x_k of the associated vector $\underline{x}$ and the variables μ_k and σ. By way of

example, the second element $\underline{y}_1(n=2)$ of the first vector $\underline{y}_1$ is calculated below:

$$\underline{y}_1(n=2) \quad = \underline{x}_1(n=2) \; * \; \sigma \; + \; \mu_1 \qquad =$$

5
$$= 0.39... \; * \; 0.45... \; - \; 0.07... \; =$$

$$= 0.10... \; .$$

With the aid of Figs. 5A-5F, there is shown a calculation example for a third simulation time step $[t_2, t_3] = [0.75, 1.25]$.
10 The value n is always equal to 3 during the third simulation time step.

Fig. 5A shows the covariance matrix $\underline{C}$ of dimension $(n \times n) = 3 \times 3$, which is required for generating one further random
15 number each per noise source. Together with the result in accordance with Fig. 4F, the random number thus newly generated forms a vector $\underline{y}_k$ of length 3 from 1/f-distributed random numbers. A vector $\underline{y}_k$ is generated in this case for each noise source. The covariance matrix $\underline{C}$ is determined in this
20 case according to equation (3.6).

For example, this is carried out at the element $\underline{C}(3,1)$ - that is to say with i=3 and j=1. From applying equation (3.6), $\underline{C}(3,1)$ is yielded as

25

$$1.0 \cdot \left(-\left| t_1 - t_3 \right|^{0.5+1} + \left| t_{1-1} - t_3 \right|^{0.5+1} + \left| t_1 - t_{3-1} \right|^{0.5+1} - \left| t_{1-1} - t_{3-1} \right|^{0.5+1} \right) =$$

$$= \left(-\left| 0.5 - 1.25 \right|^{0.5+1} + \left| 0 - 1.25 \right|^{0.5+1} + \left| 0.5 - 0.75 \right|^{0.5+1} - \left| 0 - 0.75 \right|^{0.5+1} \right) =$$

$$-0.6495... \quad + \quad 1.3975... \quad + \quad 0.125 \quad - \quad 0.6495... \quad = 0.22....$$

Fig. 5B shows the inverse $\underline{\underline{C}}^{-1}$ of the covariance matrix $\underline{\underline{C}}$ from Fig. 5A. A check (not illustrated here) of the condition ($\underline{\underline{C}}$ $\underline{\underline{C}}^-$

5 ^{1}) yields a matrix of dimension 3 × 3, in the case of which the elements indexed by (1,1), (2,2) and (3,3) are equal to 1. The other elements have the value 0.

Fig. 5C shows a variable σ that is calculated from the

10 inverted covariance matrix $\underline{\underline{C}}^{-1}$ of step 5B. The variable σ is yielded as

σ = sqrt(1 / e(n,n)) = sqrt(1 / e(3,3)) =

= sqrt(1 / 1.75...) = 0.75...;

15

sqrt denoting the square root, and e(3,3) denoting the element, indexed by (3,3) of the inverted covariance matrix $\underline{\underline{C}}^{-1}$ from Fig. 5B.

20 Fig. 5D shows three vectors $\underline{x}_k$ of independent (0,1) normally distributed random numbers, the vectors $\underline{x}_k$ each having a length of 3. A (0,1) normally distributed random variable x_k is extracted per noise source to be simulated. The extracted

random number in each case has the expectation of 0 and the

variance of 1. The vectors $\underline{x}_k$ of the normally distributed

random numbers from Fig. 4D are thereby supplemented such that

the vectors $\underline{x}_k$ of the normally distributed random numbers from

5   Fig. 5D result.

Fig. 5E shows three variables μ_k that were calculated from the

inverted covariance matrix $\underline{\underline{C}}^{-1}$ in accordance with step 5b and

from the three random numbers in accordance with step 4f. For

10   each noise source to be simulated, the variable μ_k is

calculated from the (n-1) first components of the nth row of

the inverted covariance matrix $\underline{\underline{C}}^{-1}$ and from the sequence of (n-

1) 1/f-distributed random numbers that were calculated in

accordance with formula (3.9) for the preceding (n-1)

15   simulation time steps. In the second simulation step, the

variable μ_k is thus calculated from the first two components of

the third row of $\underline{\underline{C}}^{-1}$ as well as from the first two components

of the vector $\underline{y}_k$. This is carried out, for example, with the

aid of the value μ_1:

20

$$\mu_1 = \frac{\underline{y}_{(n-1),1}^{T} \cdot \underline{\underline{C}}_{\bullet,n}^{-1}}{\underline{\underline{C}}_{n,n}^{-1}} =$$

$$= \frac{-0.30... \cdot -0.31... + 0.10... \cdot -0.98}{1.75...} =$$

$$= -0.00....$$

Fig. 5F shows three vectors $\underline{y}_k$ of length 3 with 1/f-distributed random numbers that simulate the behavior of three 1/f-distributed noise sources for the third simulation time step $[t_2, t_3] = [0.75, 1.25]$. The matrix NOISE is yielded from the

5 three vectors $\underline{y}_k$. The value k is an integral value of from 1 to 3 in this case. Each element y_k of the third row of NOISE is determined on the basis of equations (3.7)-(3.9) according to the following rule from the last random number $x_k(n=3)$ of the associated vector $\underline{x}$ and the variables μ_k and σ. By way of

10 example, the third element $\underline{y}_1(n=3)$ of the first vector $\underline{y}_1$ is calculated below:

$$\underline{y}_1(n=3) \quad = \underline{x}_1(n=3) \ * \ \sigma \ + \ \mu_1 \qquad =$$
$$= -0.90... \ * \ 0.75... \ + \ 0.00... \ =$$

15 $$= 0.67... \ .$$

The following conditions remain to be observed for the purpose of concrete execution of the calculation examples shown.

20 The numerical values shown in Figs. 3, 4 and 5 reproduce intermediate and final results of the computing steps, described with reference to Fig. 2, for a first, for a second and for a third simulation interval. In this case, after accurate numerical calculation, all values are truncated after

25 the second decimal place following the point in order to be

able to reproduce them more effectively. Consequently, when computationally reaccomplishing the exemplary embodiments, it is necessary to calculate further not with the intermediate values shown in the figures, but with the exact intermediate

5   values, in order to reach the specified $\underline{y}$ vectors starting from the specified x vectors.

Vectors of (0,1) normally distributed random variables are shown in Figs. 3C, 4C and 5C. In this case, a random variable

10   represents a noise source in each case. For the sake of simplicity, it is not illustrated here how such random numbers are reached with the aid of the expectation of 0 and the variance of 1. This is familiar to the person skilled in the art.